Karlsruhe Institute of Technology

MIMO Basics

Prof. Dr.-Ing. Thomas Zwick

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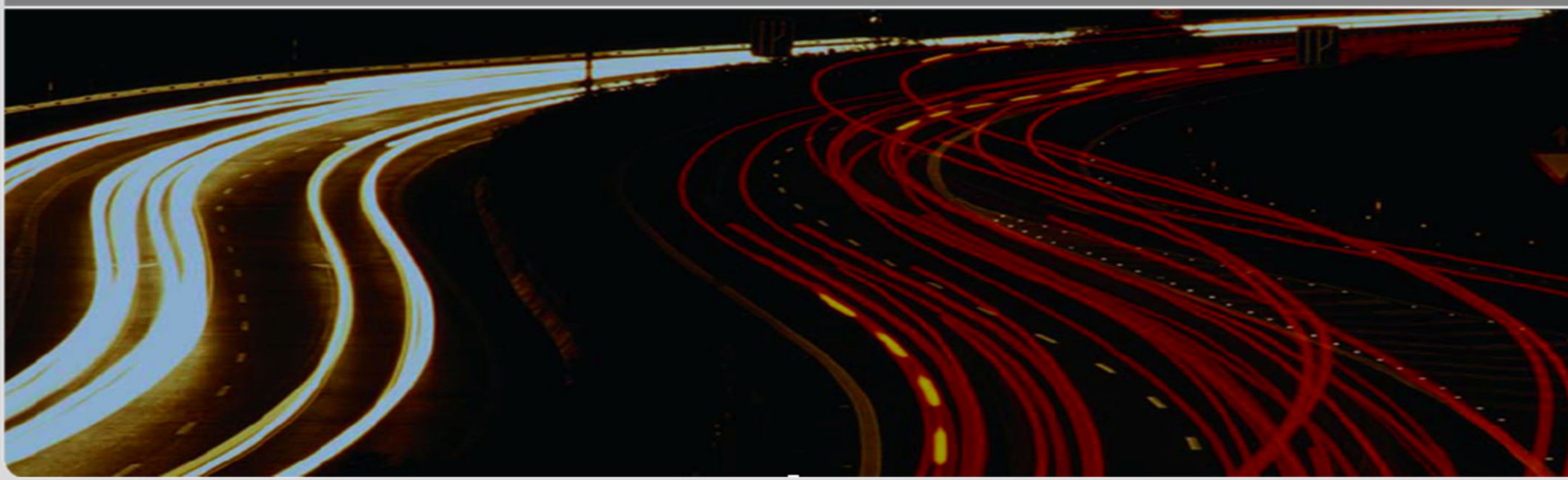

Shannon Limit 1

- The rate at which the bits may be transmitted cannot be arbitrarily increased and obeys a fundamental limit
  - Information capacity: $$C = 2B \cdot \log_2 M$$ [bit/sec]

    Channel bandwidth (B)

    Number of signal levels transmitted (M)

  - Bandwidth efficiency: $$C / B = 2 \cdot \log_2 M$$ [bit/sec/Hz]
  - For 2 levels: $$C / B = 2$$ Maximum possible to achieve with 2 levels

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Shannon Limit 2

$$C = 2B \cdot \log_2 M$$ [bit/sec]

- SISO: Information capacity $\uparrow$ if B $\uparrow$ or M $\uparrow$

- Noise added in real world: $$C = B \cdot \log_2(1 + SNR)$$ [bit/sec]

- SNR is defined as:

$$SNR = \frac{E_b R}{N_0 B}$$

$$\frac{C}{B} = \log_2\left(1 + \frac{E_b}{N_0} \cdot \frac{R}{B}\right)$$

- In practice the bit rate is lower than maximal capacity to ensure acceptable error rate

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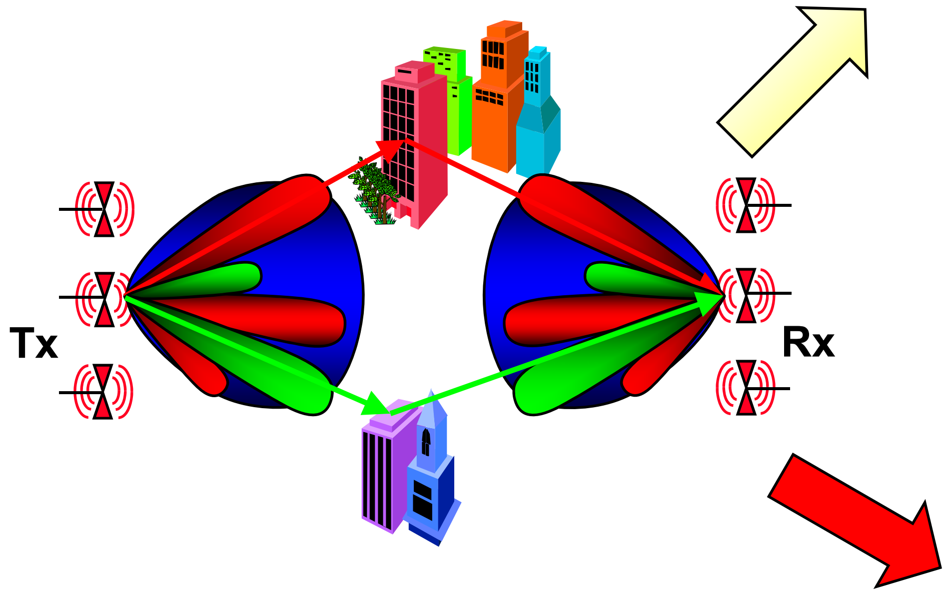
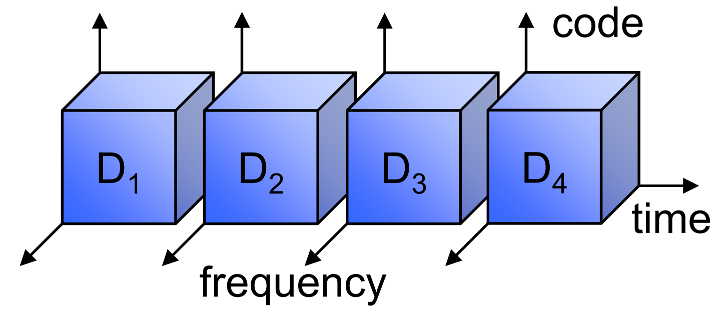
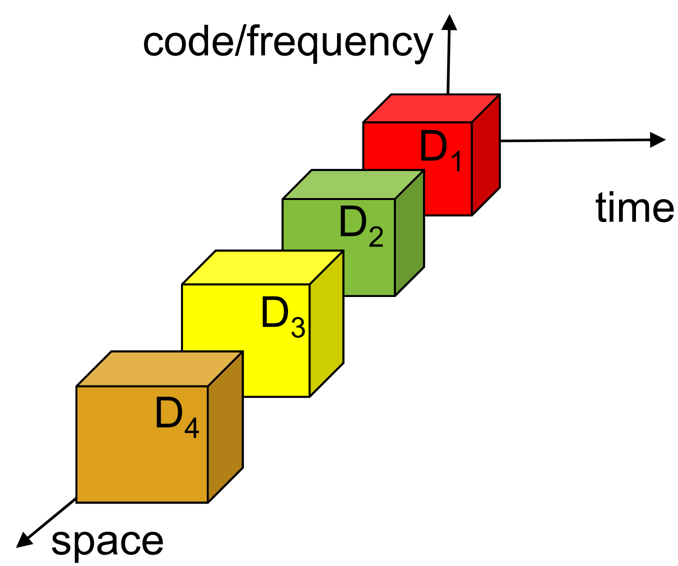
Beam-forming and Spatial Multiplexing



Beam-forming and diversity:



Spatial multiplexing



Spatial multiplexing splits the transmit signal into multiple sub-signals, and transmits each of them over a different beam characteristic or over a different sub-channel.

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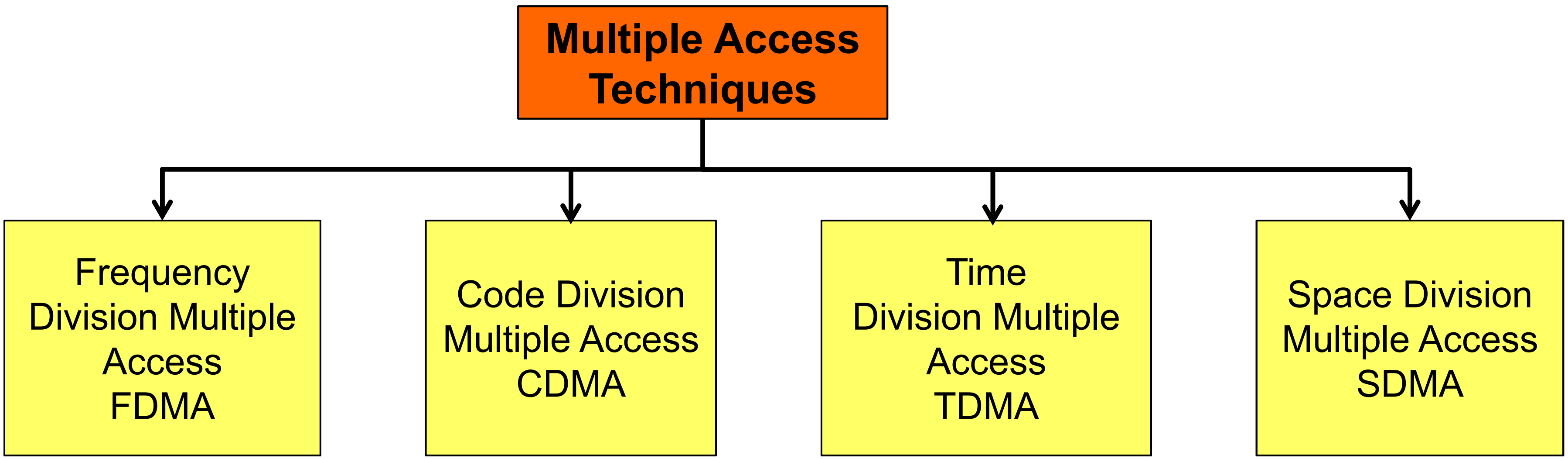
Multiple Access Techniques

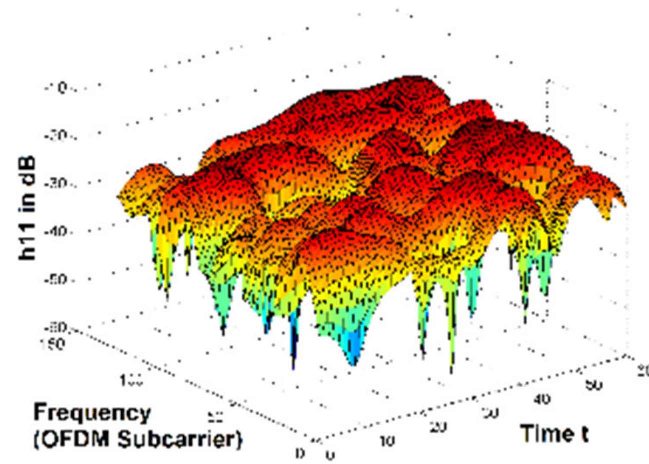


Channel Capacity of random MIMO Channels

- In general MIMO channels are not deterministic
- MIMO channels change randomly with time, space, weather, ...

$$H = H(f,t)$$

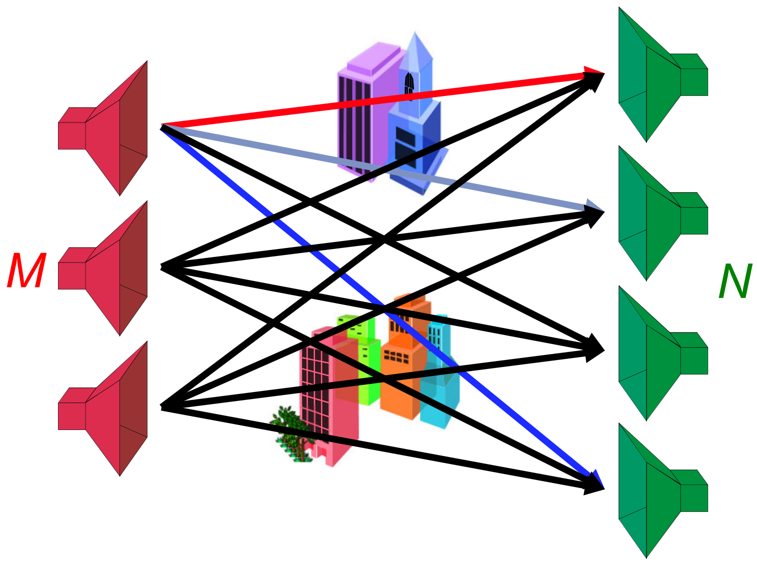
- H becomes a random matrix
- Channel capacity C is also varying randomly



$$C = C(t)$$

- The link is considered stationary and ergodic (independent behaviour)
- Real channels are non-ergodic

Channel Matrix H



$$\mathbf{H} = \begin{pmatrix} h_{11} & h_{12} & \cdots & h_{1M} \\ h_{21} & h_{22} & \cdots & h_{2M} \\ \vdots & & \ddots & \vdots \\ h_{N1} & \cdots & & h_{NM} \end{pmatrix}$$

capacity

$$C = C(\mathbf{H}, SNR)$$

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Channel-State Information (CSI)

- MIMO-Algorithms can be categorized by the amount of Channel-State Information (CSI) that they require. The following cases are distinguished:

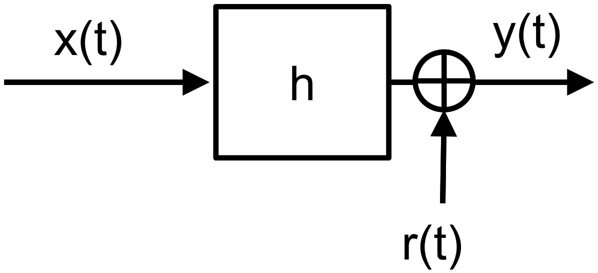
  - **Full CSI at the Tx and full CSI at the Rx:**
    In this ideal case, both the Tx and the Rx have full and perfect knowledge of the channel
    - Highest possible transinformation = capacity
    - Difficult to obtain full CSI at the Tx

  - **Average CSI at Tx and full CSI at the Rx:**
    In this ideal case, the Rx has full information of the instantaneous channel state, but the Tx knows only the average CSI (i.e. the correlation matrix of the channel impulse response or the angular power spectrum)
    - Easier to achieve
    - Reciprocity or fast feedback not required
    - Requires calibration or slow feedback

Wired SISO Capacity

- Consideration of a SISO (single-input, single-output) channel represented by a simple transmission line
  - Assumption of a pass-band communication system with narrowband signals
  - Received data y(t) is affected by AWGN r(t) with variance σ^2
  - Signal at the receiver

$$y(t) = h \cdot x(t) + r(t)$$



- Channel capacity in bit/s/Hz

$$C = \log_2(1 + SNR) = \log_2(1 + P_{Tx} \cdot \frac{|h|^2}{\sigma^2}) = \log_2(1 + \frac{P_{Rx}}{\sigma^2})$$

P_{Tx}	total transmit power
P_{Rx}	total received power
h	complex transfer factor

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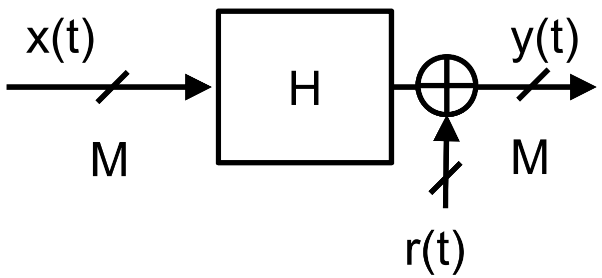
Wired MIMO Capacity

- Let us now consider a cable consisting of M transmission lines assuming that no mutual coupling exists among the transmission lines

$$\vec{y}(t) = H \cdot \vec{x}(t) + \vec{r}(t)$$

$$\vec{\mathbf{y}} = (y_{1,} y_2, \ldots, y_M)^T$$
$$\vec{\mathbf{x}} = (x_{1,} x_2, \ldots, x_M)^T$$
$$\vec{r} = (r_{1,} r_2, \ldots, r_M)^T$$

$$H = \begin{pmatrix} h_{11} & 0 & .. & 0 \\ 0 & h_{22} & 0 & 0 \\ .. & .. & .. & 0 \\ 0 & 0 & .. & h_{MM} \end{pmatrix}$$



- Further assumption for the following derivations:
  - overall transmitted power is constrained to P_{Tx}
  - transmitter has no knowledge about the channel (CSI)
  - distribute the available power equally among all M channels

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Theoretical MIMO Capacity

- The input power to each channel is P_{Tx}/M
- The channel capacity is the sum of the channel capacities of each SISO channel:

$$C = \sum_{i=1}^{M} \log_2\left(1 + \frac{P_{Tx}}{M}\frac{|h_{ii}|^2}{\sigma^2}\right) = \sum_{i=1}^{M} \log_2(1 + \mathrm{SNR}_i)$$

- C = maximum achievable data rate that can be sent
with an arbitrarily small detection error, when there is no CSI at Tx
- If all $|h_{ii}|$ are equal, the capacity increases almost linearly with M.

Capacity: SISO versus MIMO without CSI

Claude Elwood Shannon

SISO:

Classical Shannon formula

$$C = \log_2(1 + \mathrm{SNR})$$

3 dB increase in Signal to Noise Ratio gives another bit/s/Hz efficiency

MIMO:

$$C = \sum_{i=1}^{M} \log_2\left(1 + \frac{P_{Txi}}{\sigma^2}\lambda_i\right)$$

For $M_{Tx} = N_{Rx} = M$ antennas capacity grows linearly with large M

SNR Signal to Noise Ratio; H complex transmission coefficient; * compl. conjugate transposed

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Normalization of the MIMO channel matrix

- Channel capacity formula without normalization

$$C = \log_2\left(\det\left[\mathbf{I}_N + \frac{P_{Tx}}{M\sigma^2}\cdot\mathbf{H}\mathbf{H}^\dagger\right]\right)$$

- Normalization to unit Frobenius-Norm

$$\mathbf{H}_F = \frac{\mathbf{H}}{\|\mathbf{H}\|_{\mathrm{Fro}}} \qquad \text{with} \qquad \|\mathbf{H}_F\|_{\mathrm{Fro}} = 1$$

Frobenius-Norm of the MIMO channel matrix

$$\|\mathbf{H}\|_{\mathrm{Fro}} = \sqrt{\sum_{n=1}^{N}\sum_{m=1}^{M}|h_{nm}|^2}$$

- Channel capacity formula with normalization

$$C = \log_2\left(\det\left[\mathbf{I}_N + \mathrm{SNR}\cdot\mathbf{H}_F{\mathbf{H}_F}^\dagger\right]\right) \qquad \text{with} \qquad \mathrm{SNR} = \frac{P_{Tx}}{M\sigma^2}\cdot\|\mathbf{H}\|_{\mathrm{Fro}}{}^2$$

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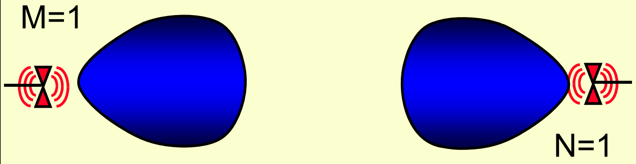
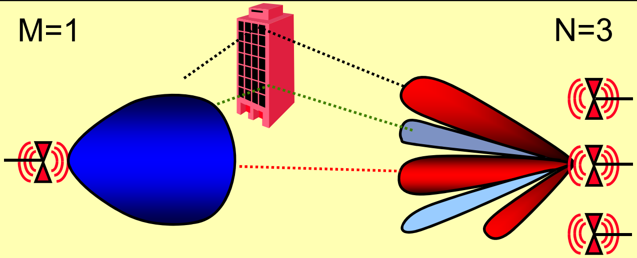
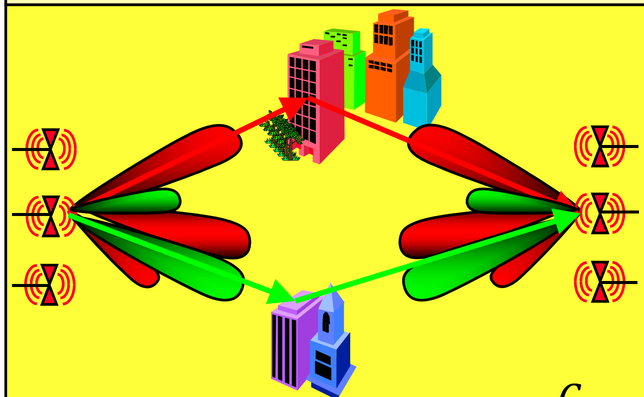
MIMO Capacity with and without CSI



$$C = \log_2\left(1 + \frac{P_{Tx}}{\sigma^2}|h_{ii}|^2\right) = \log_2(1 + \mathrm{SNR})$$



$$C = \log_2\left(\det\left[\mathbf{I}_N + \mathrm{SNR} \cdot \mathbf{h}_F \mathbf{h}_F^{\dagger}\right]\right)$$



$$C_{CSI@Tx\&Rx} = \sum_{i=1}^{N} \log_2\left(1 + \frac{P_{Tx,i}}{\sigma^2} \cdot \lambda_i\right)$$

summation of the sub-channel capacities

with channel state information (CSI) at Tx

$$C_{\mathrm{CSI@Rx}} = \log_2\left(\det\left[\mathbf{I}_N + \mathrm{SNR} \cdot \mathbf{H}_F \mathbf{H}_F^{\dagger}\right]\right)$$

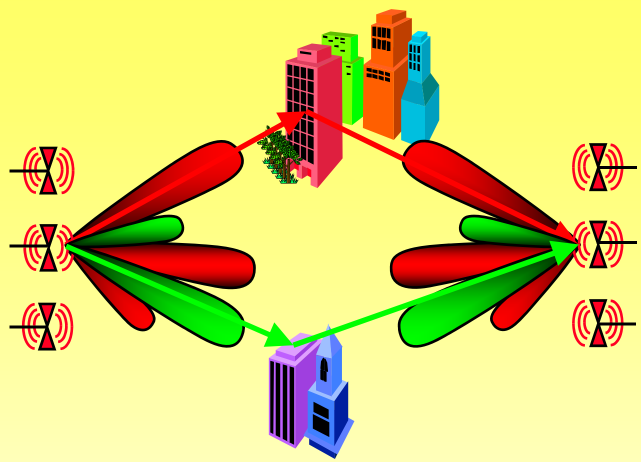
without channel state information (CSI) at Tx

P_{Txi} sub-channel *i* transmit power
λ_i sub-channel *Eigen value*
σ^2 noise power

h = channel impulse response
h = channel impulse response vector
H = MIMO channel matrix

H⁺ = hermitian matrix

MIMO Channel Capacity with CSI at Transmitter

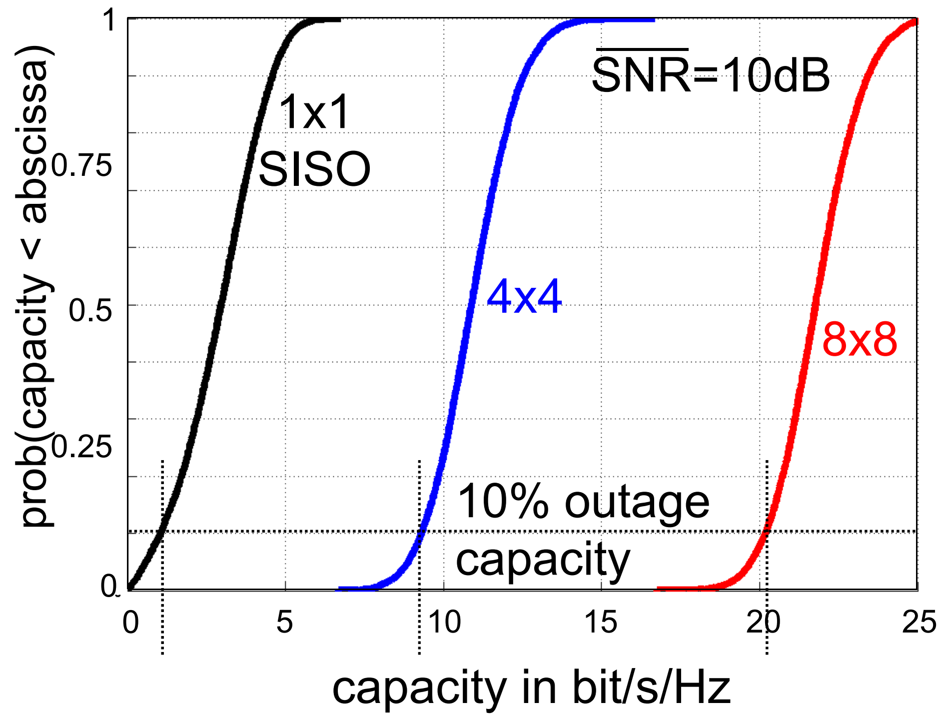


summation of the capacities
of the n sub-channels

$$C_{MIMO} = \sum_{i=1}^{n} \log_2 \left(1 + \frac{P_{Tx,i} \lambda_i}{\sigma^2} \right) \frac{\text{bit}}{\text{s} \cdot \text{Hz}}$$

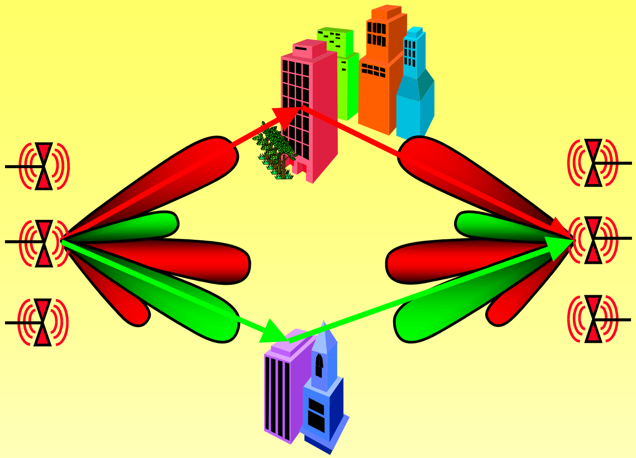
P_{Txi} transmit power in channel i
λ_{i} sub-channel *Eigen value*
σ^2 noise power

n = min(M, N, rank of channel matrix)



The x% outage capacity is the capacity value that yields an undershoot-probability of x%.

MIMO Channel Capacity with CSI at Transmitter



summation of the capacities
of the n sub-channels

$$C_{MIMO} = \sum_{i=1}^{n} \log_2\left(1+\frac{P_{Tx,i}\lambda_i}{\sigma^2}\right)\frac{\mathrm{bit}}{\mathrm{s\cdot Hz}}$$

P_{Txi} transmit power in channel i
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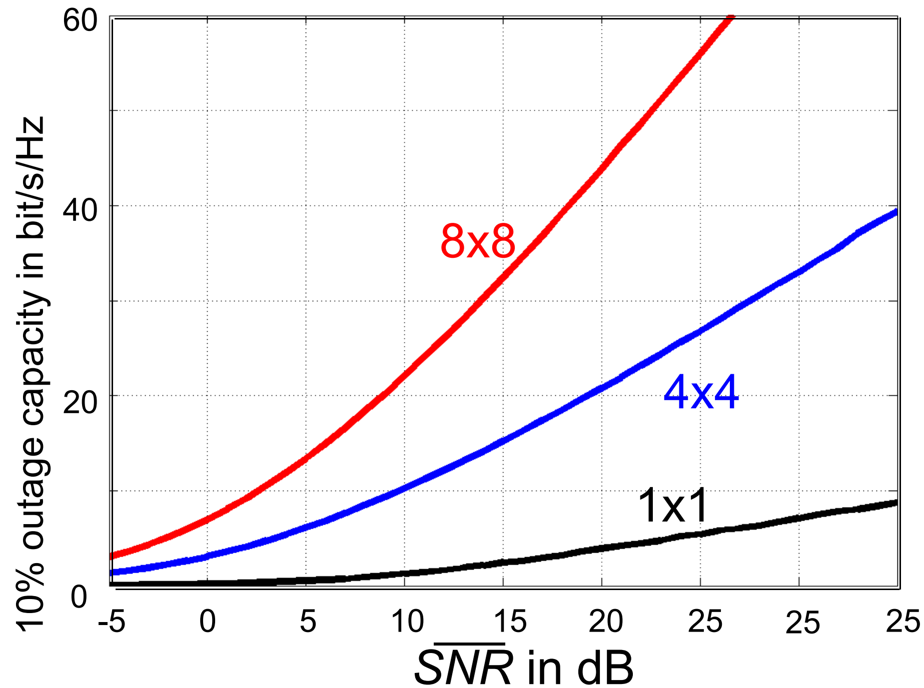
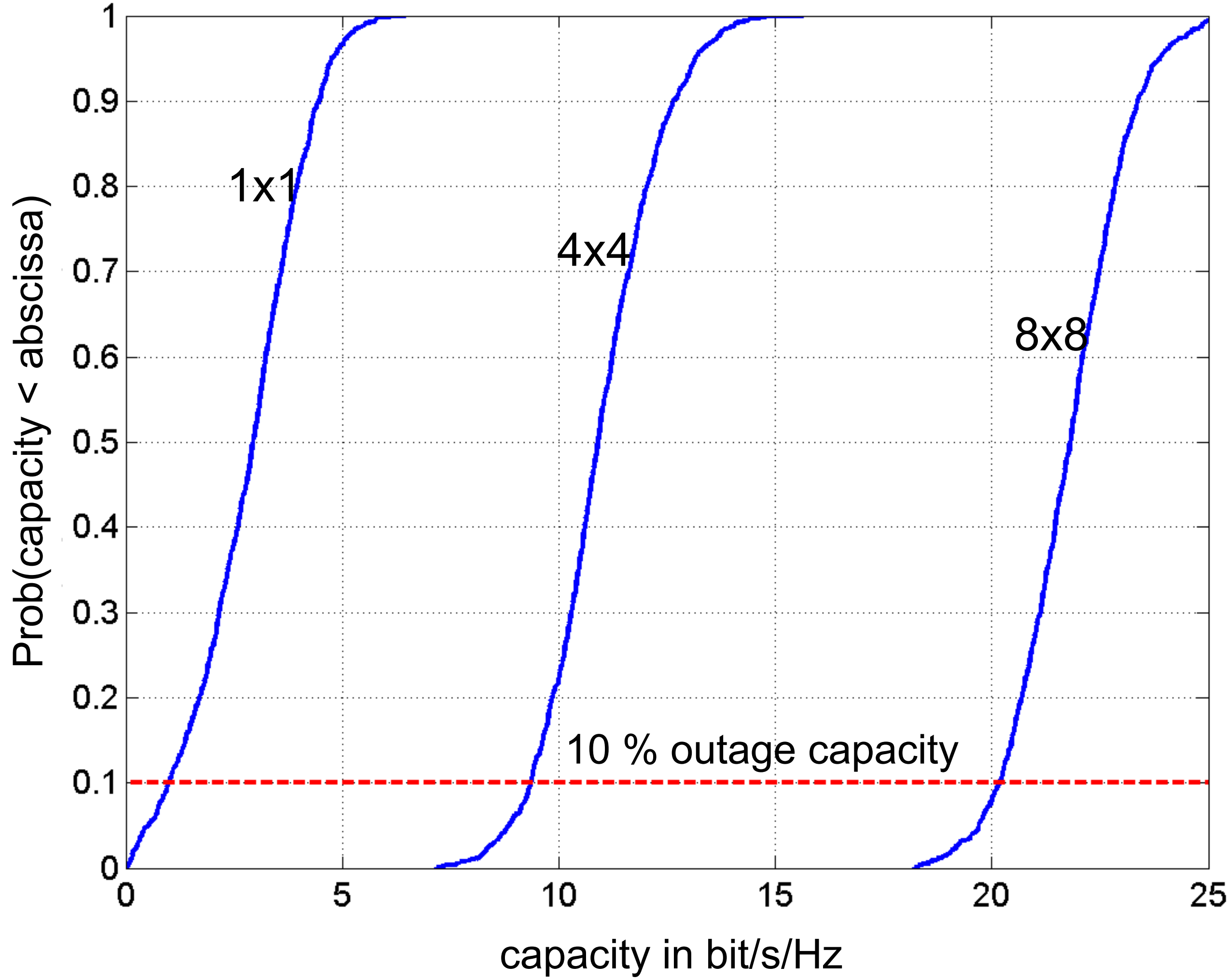


MIMO: large potential for future high data rate communications!

CDF-Plots for MIMO Capacities

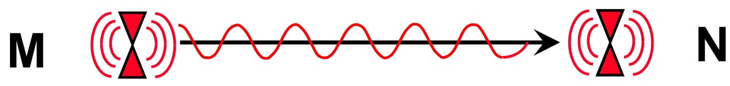
Constant SNR=10dB

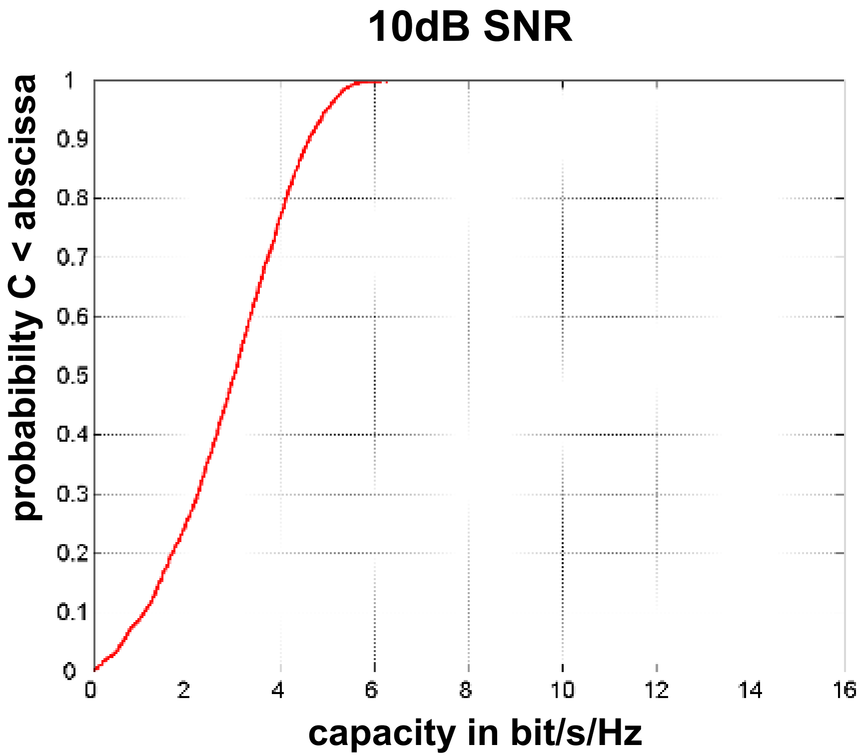


Capacity of MIMO Systems

SISO:

$$C = \log_2(1 + \mathrm{SNR})$$





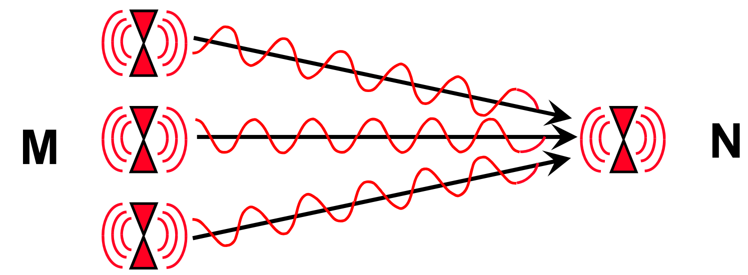
Capacity of MIMO Systems

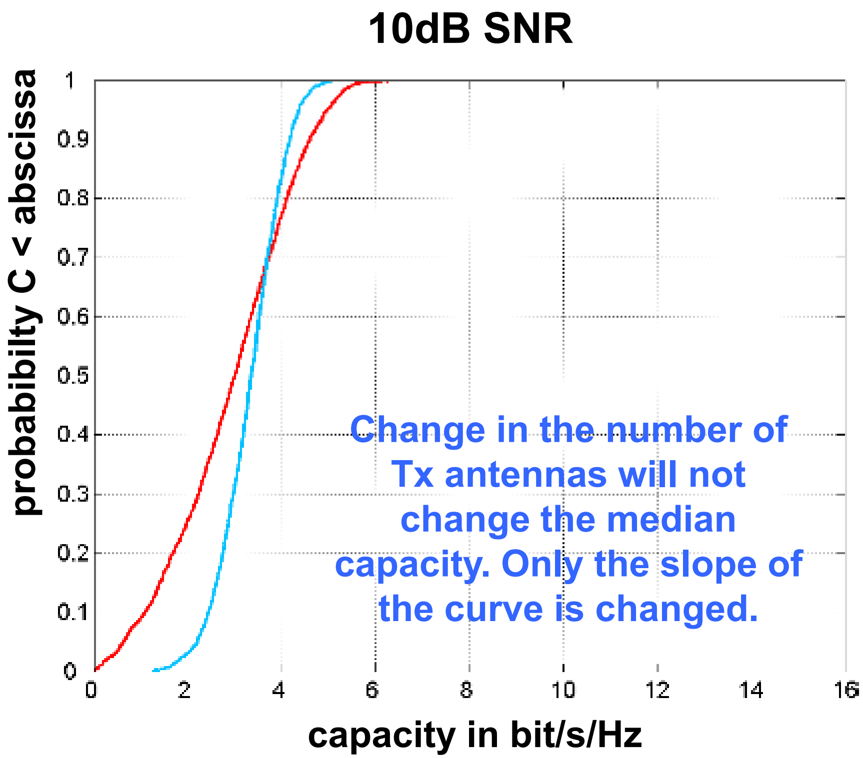
SISO:

$$C = \log_2(1 + \mathrm{SNR})$$

MISO:

$$C = \log_2\left(1 + \mathrm{SNR} \cdot \mathbf{h}_F \mathbf{h}_F^{\dagger}\right)$$





$H^{+} = (H^{T})^{*} = (H^{*})^{T}$ = hermitian matrix
= complex conjugate transposed matrix

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Capacity of MIMO Systems

SISO:

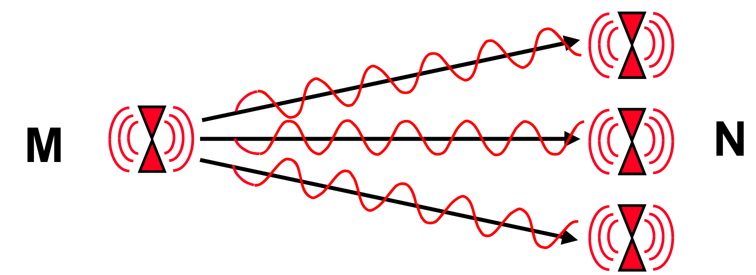
$$C = \log_2(1 + \mathrm{SNR})$$

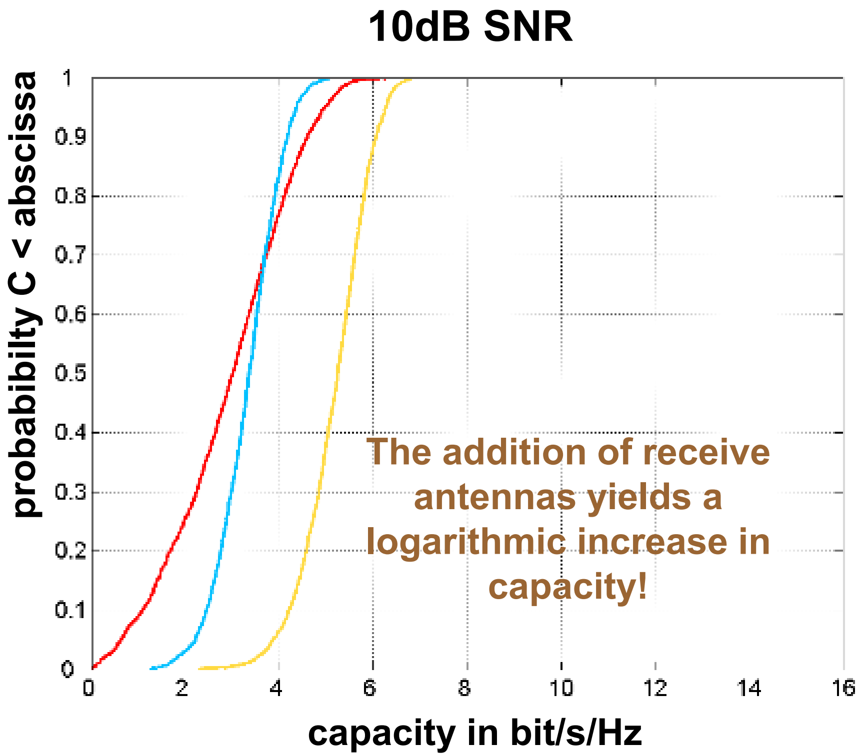
MISO:

$$C = \log_2\left(1 + \mathrm{SNR} \cdot \mathbf{h}_F \mathbf{h}_F^{\dagger}\right)$$

SIMO:

$$C = \log_2\left(\det\left[\mathbf{I}_N + \mathrm{SNR} \cdot \mathbf{h}_F \mathbf{h}_F^{\dagger}\right]\right)$$





$H^{+} = (H^{T})^{*} = (H^{*})^{T}$ = hermitian matrix

= complex conjugate transposed matrix

Capacity of MIMO Systems

SISO:

$$C = \log_2(1 + \mathrm{SNR})$$

MISO:

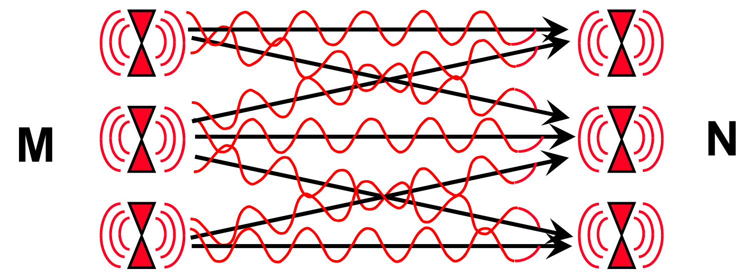
$$C = \log_2\left(1 + \mathrm{SNR} \cdot \mathbf{h}_F \mathbf{h}_F^{\dagger}\right)$$

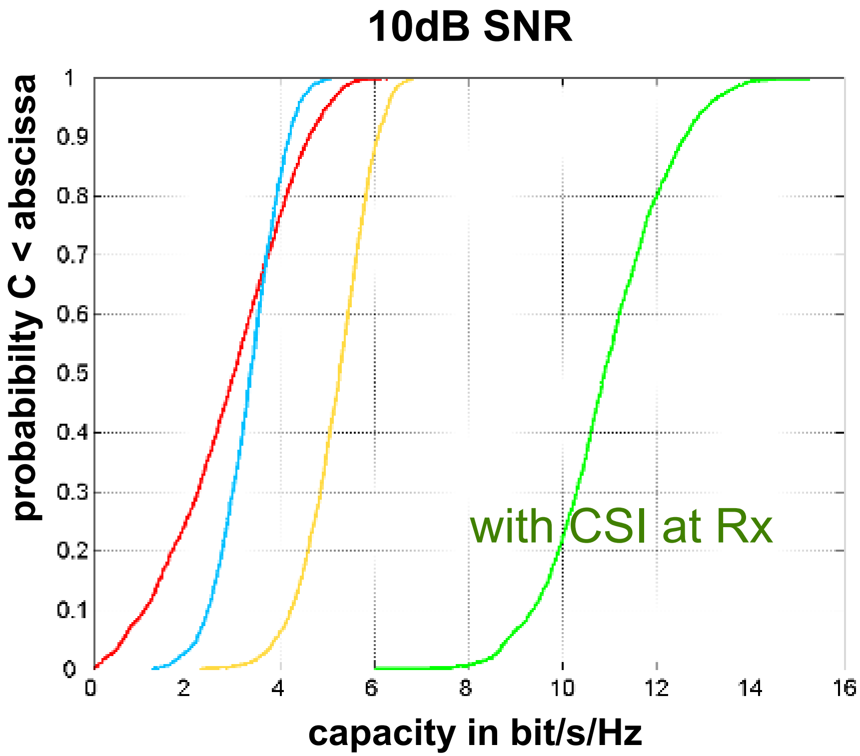
SIMO:

$$C = \log_2\left(\det\left[\mathbf{I}_N + \mathrm{SNR} \cdot \mathbf{h}_F \mathbf{h}_F^{\dagger}\right]\right)$$

MIMO:

$$C = \log_2\left(\det\left[\mathbf{I}_N + \mathrm{SNR} \cdot \mathbf{H}_F \mathbf{H}_F^{\dagger}\right]\right)$$





$H^+ = (H^T)^* = (H^*)^T$ = hermitian matrix
= complex conjugate transposed matrix

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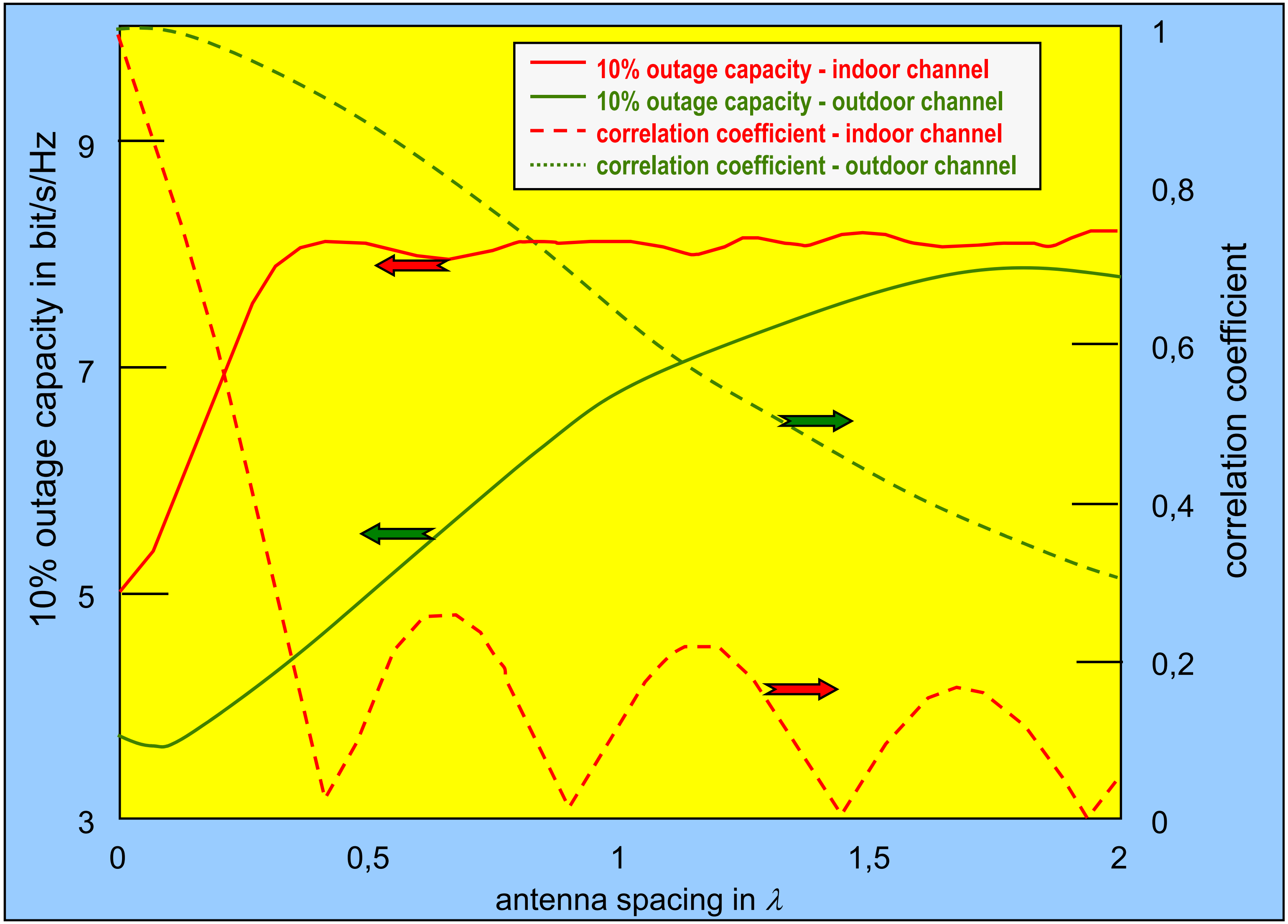
Influence of the Signal Correlation



Values are based on exemplary simuation setups.

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MIMO Classification

CSI: channel state information

MIMO
multiple input multiple output

Beamforming based MIMO
only best subchannel

- lower complexity
- long term channel statistics stable
- optimum for small SNR and interference

Multiplex Transmission
several subchannels

CSI at Tx

Waterfilling

- optimal MIMO system
- instantanious CSI
- beamforming for each subchannel

no CSI at Tx

equal Tx power/antenna
BLAST
Space Time Codes

- high capacity
- not too complex
- already realizable

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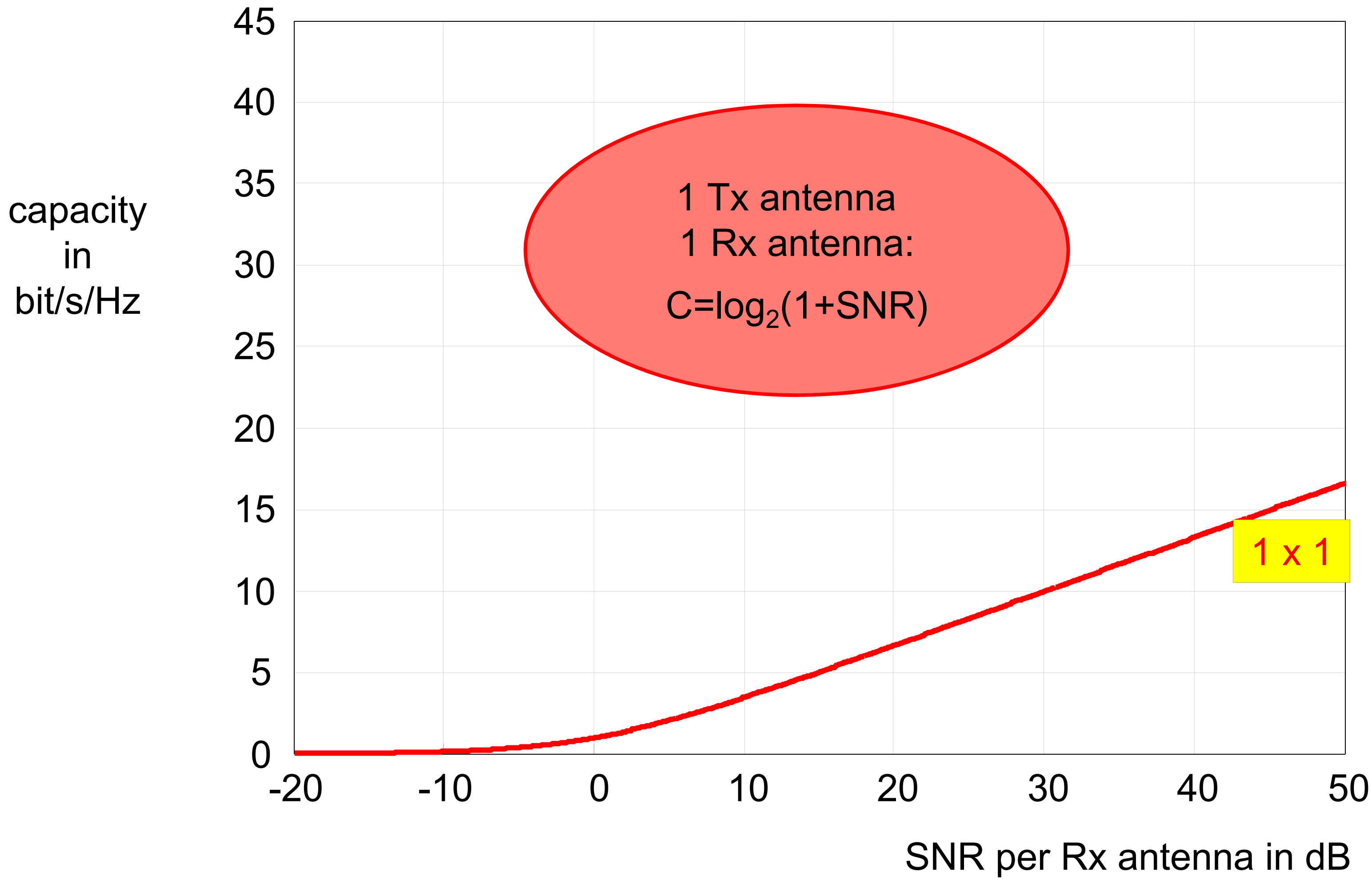
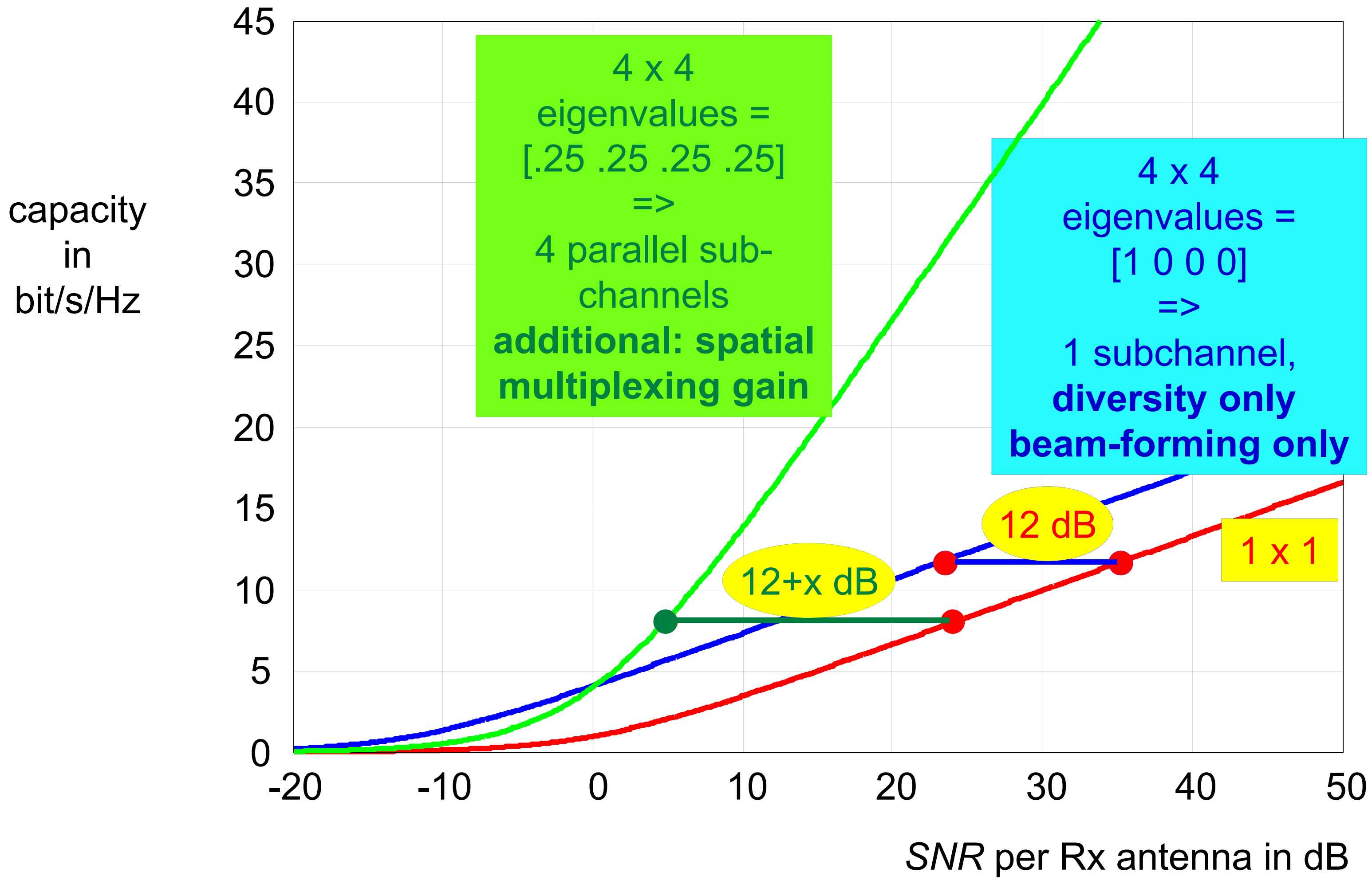
MIMO Capacities: 1 x 1 Channel



MIMO capacities : 4 x 4 versus 1 x 1 channels (1)



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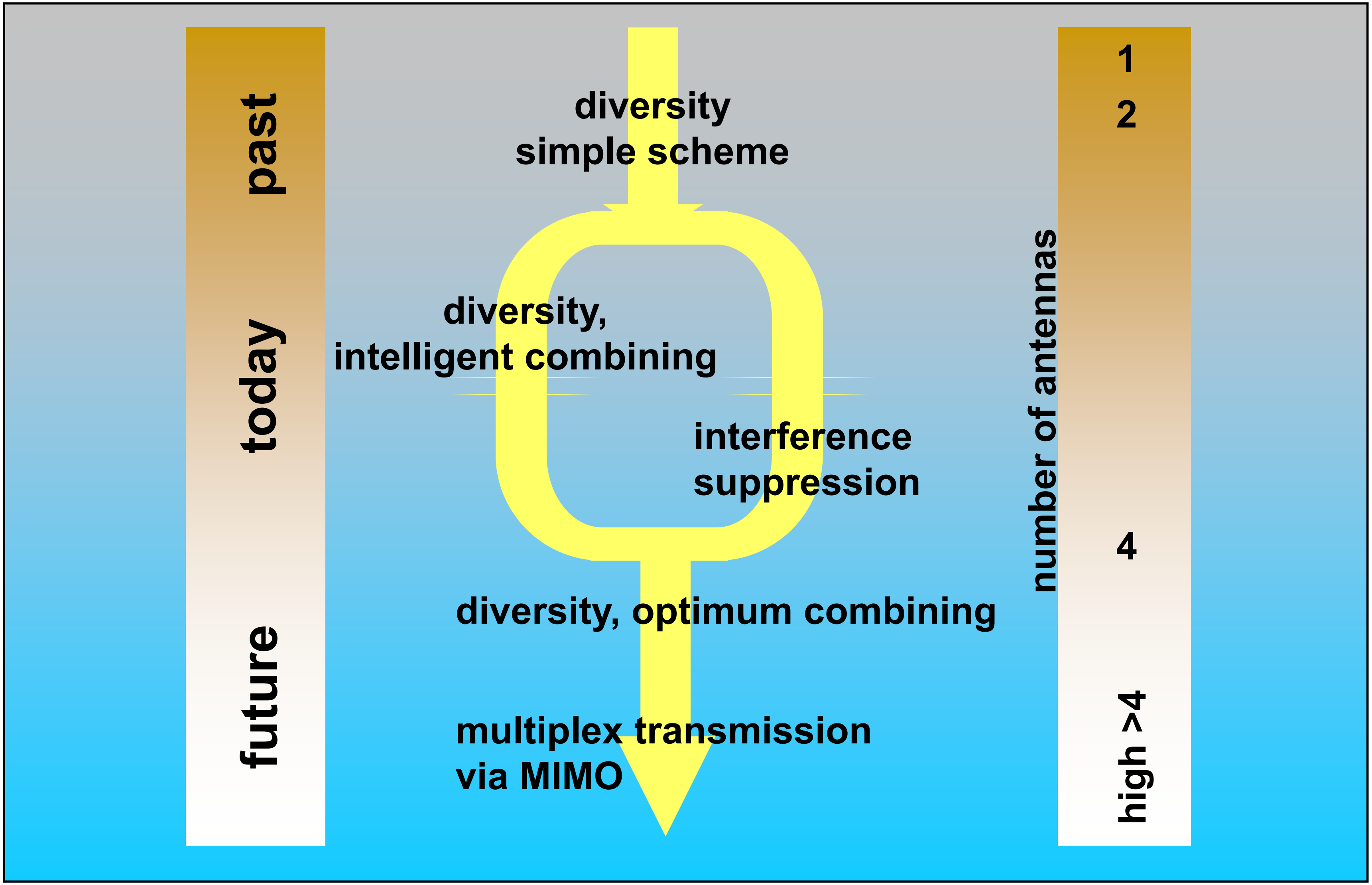
Evolution of Multi-Antenna Systems



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References

(1) C. F. Mecklenbräuker, A. F. Molisch, J. Karedal, F. Tufvesson, A. Paier, L. Bemado and T. Zemen, O. Klemp, and N. Czink , "Vehicular channel characterization and its implications for wireless system design and performance," Proceedings of the IEEE, vol. 99. no. 7, pp. 1189-1212. 2011.

(2) Molisch, F. Tufvessoll, J. Karedal, and C. Mecklenbräuker. "Propagation aspects of vehicle-to-vehicle communications - an overview." in IEEE Radio and Wireless Symposium, Jan. 2009, pp. 179- 182.

(3) A. Molisch, F. Tufvesson, J. Karedal, and C. Mecklenbräuker, "A Survey on vehicle-to-vehicle propagation channels," IEEE Wireless Communications, vol. 16, no. 6, pp. 12-22, Dec . 2009.

(4) L. Reichardt, J. Pontes, W. Wiesbeck, and T. Zwick. "Virtual drives in vehicular communications," IEEE Vehicular Technology Magazine, vol. 6, no. 2, pp. 54-62, June 2011.

(5) L. Reichardt, J. Maurer, T. Fügen, and T. Zwick "Virtual Drive: A complete V2X communication and radar system simulator for optimization of multiple antenna systems." Proceedings of the IEEE, vol. 99, no. 7, pp. 1295-1310, July 2011 .

(6) J. Pontes. "Optimized analysis and design of multiple element antennas for urban communication”, Ph.D. dissertation, Karlsruhe Inst. Technology (KIT). 2010.

(7) J. Pontes and T. Zwick, "Antenna synthesis for multiple element antenna channels” in Proc. European Conference on Antennas and Propagation (EuCAP), Apr. 2010, pp. 1- 3.

(8) L. Reichardt, J . Pontes, G. Jereczek and T. Zwick. "Capacity maximizing MIMO antenna design for car-to-car communication” in Proc. International Workshop on Antenna Technology (iWAT), Mar. 2011, pp. 243- 246.

(9) L. Reichardt. C. Sturm. and T. Zwick. "Performance evaluation of SISO, SIMO and MIMO antenna systems for car-to-car communications in urban environments” in Proc. International Conference on Intelligent Transport Systems Telecommunications (ITST), Oct. 2009, pp. 51 - 56.

(10)L. Reichardt, T. Fügen, and T. Zwick, "Influence of antennas placement on car to car communications channel," in Proc. of the European Conference on Antennas and Propagation (EuCAP), Mar. 2009, pp. 630-634.

(11)D. Kornek, M. Schack. E. Slottke, O. Klemp, I. Rolfes, and T. Kürner, "Effects of antenna characteristics and placements on a vehicle-to-vehicle channel scenario”· in Proc. IEEE International Conference on Communications Workshops (ICC), May 2010, pp. 1- 5.

(12)O. Klemp, "Performance considerations for automotive antenna equipment in vehicle-to-vehicle communications” in Proc. URSI International Symposium on Electromagnetic Theory (EMTS), Aug. 2010, pp. 934-937.